Chapter 6
Investment Decision Rules

6-1. Timeline:

\[
\begin{array}{cccccccc}
& 0 & 1 & 2 & 3 & 4 \\
\hline
-100 & 30 & 30 & 30 & \cdots \\
\end{array}
\]

\[\text{NPV} = \left( \frac{1}{1.08} \right)^{30} \frac{30}{0.08} - 100 = $247.22 \text{ million} \]

The IRR solves

\[
\left( \frac{1}{1 + r} \right)^{30} \frac{30}{r} - 100 = 0 \Rightarrow r = 24.16\%
\]

So, the cost of capital can be underestimated by 16.16% without changing the decision.

6-2. a. Timeline:

\[
\begin{array}{cccc}
& 0 & 1 & 2 \\
\hline
10 & -8 & -8 \\
\end{array}
\]

\[\text{NPV} = 10 - \frac{8}{0.1} \left(1 - \frac{1}{(1.1)^3}\right) = -$9.895 \text{ million} \]

b. Timeline:

\[
\begin{array}{cccccccc}
& 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
10 & -8 & -8 & -8 & 5 & 5(1 - 0.3) & 5(1.03)^2 & \cdots \\
\end{array}
\]

First calculate the PV of the royalties at year 3. The royalties are a declining perpetuity:

\[\text{PV}_5 = \frac{5}{0.1 - (-0.3)} = \frac{5}{0.4} = 12.5 \text{ million} \]
So the value today is

$$PV_{\text{royalties}} = \frac{12.5}{(1.1)^3} = 9.391$$

Now add this to the NPV from part a), NPV = −9.895 + 9.391 = −$503,381.

6-3.

a. Timeline:

\[
\begin{array}{ccccccccc}
0 & 1 & 2 & 3 & 6 & 7 & \cdots & 16 \\
-200,000 & -200,000 & -200,000 & -200,000 & 300,000 & 300,000 & \cdots & \\
\end{array}
\]

$$\text{NPV} = \frac{200,000}{r} \left( \frac{1}{(1+r)} \right)^0 + \left( \frac{1}{(1+r)} \right)^3 + \frac{300,000}{r} \left( \frac{1}{(1+r)} \right)^6$$

i. $$\text{NPV} = \frac{200,000}{0.1} \left( \frac{1}{(1.1)} \right)^0 + \left( \frac{1}{(1.1)} \right)^3 + \frac{300,000}{0.1} \left( \frac{1}{(1.1)} \right)^6$$

= $169,482

NPV > 0, so the company should take the project.

ii. Setting the NPV = 0 and solving for r (using a spreadsheet) the answer is IRR = 12.66%.
So if the estimate is too low by 2.66%, the decision will change from accept to reject.

iii. The new timeline is

\[
\begin{array}{ccccccccc}
0 & 1 & 2 & 3 & N & N+1 & \cdots & N+10 \\
-200,000 & -200,000 & -200,000 & -200,000 & 300,000 & 300,000 & \cdots & \\
\end{array}
\]

$$\text{NPV} = \frac{200,000}{r} \left( \frac{1}{(1+r)^N} \right) + \left( \frac{1}{(1+r)^N} \right)^3 + \frac{300,000}{r} \left( \frac{1}{(1+r)^N} \right)^6$$

Setting the NPV = 0 and solving for N gives

$$N = \frac{\log \left( \frac{500,000 - \left( \frac{300,000}{(1+r)^N} \right)}{200,000} \right)}{\log (1+r)} = \frac{\log \left( \frac{2.5 \cdot 1.5}{1.1^N} \right)}{\log (1.1)} = 6.85 \text{ years}$$
b. 

i. Timeline:

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 6 & 7 & \ldots & 16 \\
-200,000 & -200,000 & -200,000 & -200,000 & 300,000 & 300,000 & \ldots & 300,000 \\
\end{array}
\]

\[
\text{NPV} = \frac{-200,000}{r} \left(1 - \frac{1}{(1+r)^1}\right) + \frac{300,000}{1+r} \left(1 - \frac{1}{(1+r)^6}\right)
\]

\[
= \frac{-200,000}{0.14} \left(1 - \frac{1}{(1.14)^1}\right) + \frac{300,000}{0.14} \left(1 - \frac{1}{(1.14)^6}\right)
\]

\[
= -$64.816
\]

ii. Since the IRR still has not changed it is still 12.66%, so if the estimate is too high by 1.34%, the decision will change.

iii. Setting the NPV = 0 and solving for N gives:

\[
\text{NPV} = \frac{-200,000}{0.14} \left(1 - \frac{1}{(1.14)^1}\right) + \frac{300,000}{0.14} \left(1 - \frac{1}{(1.14)^6}\right) = 0
\]

\[
-777.733.5 + 976.256.9 \left(1 - \frac{1}{(1.14)^4}\right) = 0
\]

\[
= 198,523.4 \left(1 - \frac{1}{(1.14)^4}\right) = 0 \Rightarrow (1.14)^N = 4.9176
\]

\[
\Rightarrow N \log(1.14) = \log(4.9176) \Rightarrow 0.131N = 1.5928 \Rightarrow N = 12.16 \text{ years}
\]

6-4. 10 months.

6-5. Timeline:

\[
\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & \ldots \ldots \\
-100 & 30 & 30 & 30 & 30 & \ldots \ldots \\
\end{array}
\]

\[
\text{NPV} = \left(\frac{1}{1.08}\right)^{0.08} - 100 = $247.22 \text{ million}
\]

The IRR solves

\[
\left(\frac{1}{1+r}\right)^{0.08} - 100 = 0 \Rightarrow r = 24.16\%
\]

Since the IRR exceeds the 8% discount rate the IRR gives the same answer as the NPV rule.
6-6. Timeline:

\[ NPV = 0 = 10 - \frac{8}{r} \left( 1 - \frac{1}{(1 + r)^3} \right) \]

To determine how many solutions this equation has, plot the NPV as a function of \( r \)

From the plot there is one IRR of 60.74%.

Since the IRR is much greater than the discount rate the IRR rule says write the book. Since this is a negative NPV project (from 6.2a) the IRR gives the wrong answer.

6-7. Timeline:

From 6.2(b) the NPV of these cash flows is
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NPV = 10 - \( \frac{8}{r} \left( 1 - \frac{1}{(1 + r)^3} \right) + \frac{1}{(1 + r)^3} \left( \frac{5}{r + 0.3} \right) \)

Plotting the NPV as a function of the discount rate gives

The plot shows that there are 2 IRRs – 7.165% and 41.568%. The IRR does give an answer in this case, no it does not work.

6-8. The timeline of this investment opportunity is:

\[
\begin{array}{ccccccc}
0 & 1 & 2 & \cdots & 12 \\
50,000 & -4,400 & -4,400 & & -4,400 \\
\end{array}
\]

Computing the NPV of the cash flow stream

\[
NPV = 50,000 - \frac{4,400}{r} \left( 1 - \frac{1}{(1 + r)^{12}} \right)
\]

To compute the IRR, we set the NPV equal to zero and solve for \( r \). Using the annuity spreadsheet gives

<table>
<thead>
<tr>
<th>N</th>
<th>I</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>0.8484%</td>
<td>50,000</td>
<td>-4,400</td>
<td>0</td>
</tr>
</tbody>
</table>

The monthly IRR is 0.8484, so since

\[(1.008484)^{12} = 1.106696\]

0.8484% monthly corresponds to an EAR of 10.67%. Smith’s cost of capital is 15%, so according to the IRR rule, she should turn down this opportunity.
Let's see what the NPV rule says. If you invest at an EAR of 15%, then after one month you will have

\[(1.15)^{\frac{1}{12}} = 1.011715\]

so the monthly discount rate is 1.1715%. Computing the NPV using this discount rate gives

\[
\text{NPV} = 50,000 - \frac{4,400}{0.011715} \left( 1 - \frac{1}{(1.011715)^{12}} \right) = 1010.06
\]

Which is positive, so the correct decision is to accept the deal. Smith can also be relatively confident in this decision. Based on the difference between the IRR and the cost of capital, her cost of capital would have to be 15 – 10.67 = 4.33% lower to reverse the decision

6-9.

a. Timeline:

```
0 1 2    10 11 12
-5 1 - 0.1 1 - 0.1    1 - 0.1 0.1 0.1
```

The PV of the profits is

\[
\text{PV}_{\text{profits}} = \frac{1}{r} \left( 1 - \frac{1}{(1 + r)^{10}} \right)
\]

The PV of the support costs is

\[
\text{PV}_{\text{support}} = \frac{0.1}{r}
\]

\[
\text{NPV} = -5 + \text{PV}_{\text{profits}} + \text{PV}_{\text{support}} = -5 + \frac{1}{r} \left( 1 - \frac{1}{(1 + r)^{10}} \right) - \frac{0.1}{r}
\]

\(r = 5.438701\%\) then NPV = \$721,162

\(r = 2.745784\%\) then NPV = 0

\(r = 10.879183\%\) then NPV = 0

b. From the answer to part (a) there are 2 IRRs: 2.745784% and 10.879183%

c. The IRR rule says nothing in this case because there are 2 IRRs

6-10. The timeline of this investment opportunity is:

```
0 1 2    10 11 12
```

\[\boxed{}\]

\[\boxed{}\]
Computing the NPV of the cash flow stream:

\[
\text{NPV} = -120 + \frac{20}{r} \left(1 - \frac{1}{(1+r)^{10}}\right) - \frac{2}{r(1+r)^{10}}
\]

You can verify that \( r = 0.02924 \) or 0.08723 gives an NPV of zero. There are two IRRs, so you cannot apply the IRR rule. Let’s see what the NPV rule says. Using the cost of capital of 8% gives

\[
\text{NPV} = -120 + \frac{20}{r} \left(1 - \frac{1}{(1+r)^{10}}\right) - \frac{2}{r(1+r)^{10}} = 2.621791
\]

So the investment has a positive NPV of $2,621,791. In this case the NPV as a function of the discount rate is \( n \) shaped.

If the opportunity cost of capital is \( \text{between } 2.93\% \) and 8.72\%, the investment should be undertaken.

6-11. Timeline:

\[
P_{\text{operating profits}} = \frac{20}{r} \left(1 - \frac{1}{(1+r)^{20}}\right)
\]

In year 20, the PV of the stabilizations costs are \( PV_{20} = \frac{5}{r} \)

So the PV today is \( PV_{\text{stabilization costs}} = \frac{1}{(1+r)^{20}} \left(\frac{5}{r}\right)\)
NPV = -250 + \frac{20}{r} \left( 1 - \frac{1}{(1 + r)^20} \right) - \frac{1}{(1 + r)^20} \left( \frac{5}{r} \right)

Plotting this out gives

So no IRR exists.

**6-12.** Timeline:

<table>
<thead>
<tr>
<th>Date (t)</th>
<th>EVA</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-100</td>
</tr>
<tr>
<td>1</td>
<td>-100r</td>
</tr>
<tr>
<td>2</td>
<td>30 - 100r</td>
</tr>
<tr>
<td>3</td>
<td>30 - 100r</td>
</tr>
<tr>
<td>4</td>
<td>30 - 100r</td>
</tr>
</tbody>
</table>

In the first year there are no profits but capital is still being tied up, so we still need to take this cost into account. So the EVA at date 1 is

EVA\(_1\) = -100r

After that the EVA is

EVA\(_N\) = 30 - 100r

At date 1 the PV of all future EVAs is

PV\(_1\) = \frac{30 - 100r}{r}
So the PV of this today is
\[
PV_t = \left( \frac{30 - 100r}{r} \right) \left( \frac{1}{1 + r} \right) = \frac{30 - 100(0.08)}{0.08} \frac{1}{1.08} = 254.63 \text{ million}
\]

The PV of the EVA at date 1 is
\[
PV_{date 1} = \frac{-100r}{1 + r} = \frac{-100(0.08)}{1.08} = -7.41
\]

So the PV of all future EVAs is
\[
PV = PV_t + PF_{date 1} = 254.13 - 7.41 = 247.22 \text{ million}
\]

The EVA rule gives the same answer as the NPV rule.

6-13. Timeline:

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & \cdots & 12 \\
\text{Cash Flow} & -100 & 15 & 15 & \cdots & 15 + 10 \\
\text{EVA} & -100r & 15 - 100 & 15 - 100r & \cdots & 15 - 100r - (100 - 10)
\end{array}
\]

EVA in year 1 is \(-100r\) because there is cash generated, but capital is still tied up. The PV is
\[
PV_{Year 1 \ EVA} = \frac{-100r}{1 + r} = \frac{-100(0.12)}{1.12} = -10.714
\]

EVA in year 2–20 is
\[
15 - 100r
\]

Calculate the PV in year 1 gives
\[
\frac{15 - 100r}{r} \left( 1 - \frac{1}{(1 + r)^{19}} \right)
\]

So the PV today is
\[
PV_{Y2-20 \ EVA} = \left( \frac{1}{1 + r} \right) \frac{15 - 100r}{r} \left( 1 - \frac{1}{(1 + r)^{19}} \right) = \left( \frac{1}{1.12} \right) \frac{15 - 100(0.12)}{0.12} \left( 1 - \frac{1}{(1.12)^{19}} \right) = 19.73
\]

EVA in year 21, when the capital is used is
\[
15 - 100r - (100 - 10)
\]
So the PV of this today is

\[
PV_{Year \ 21 \ EVA} = \frac{15 - 100r - (100 - 10)}{(1 + r)^{21}} = \frac{15 - 100(0.12) - 90}{(1.12)^{21}} = -8.053
\]

PV of the EVAs is

\[
PV = PV_{Year \ 1 \ EVA} + PV_{Y2-20 \ EVA} + PV_{Year \ 21 \ EVA} = 6.143. \hspace{1em} \text{Take the project.}
\]

The NPV is

\[
NPV = -100 + \frac{15}{r(1 + r)} \left(1 - \frac{1}{(1 + r)^{20}}\right) + \frac{10}{(1 + r)^{21}} = 0.963
\]

So the two rules agree.

6-14. Timeline:

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & -10 \\
\hline
& & & & & & & \\
-10 & 0 & 5 & 2 & 2 & 2 & 2 & \\
\end{array}
\]

It will take 5 years to pay back the initial investment so the payback period is 4 years. You will not make the movie.

\[
NPV = -10 + \frac{5}{(1 + r)^2} + \frac{2}{r} \left(1 - \frac{1}{(1 + r)^4}\right) \frac{1}{(1 + r)^2} = -10 + \frac{5}{(1.1)^2} + \frac{2}{0.1(1.1)^2} \left(1 - \frac{1}{(1.1)^4}\right) = -628,322
\]

So the NPV agrees with the payback rule in this case.

6-15. Compute the IRR by plotting the NPV as a function of the discount rate and marking the IRR where the graph crosses the x axes. Give your boss this plot rather than the single IRR number.

6-16.

a. Timeline:

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & - & - & - & - \\
\hline
A & \hline & 2 & 2 & 2 & \\
& -10 &&&&& \\
B & \hline & 1.5 & 1.5(1.02) & 1.5(1.02)^2 & \\
& -10 &&& \\
\end{array}
\]

\[
NPV_A = \frac{2}{r} - 10
\]

Setting \(NPV_A = 0\) and solving for \(r\)
IRR_A = 20%

\[ \text{NPV}_B = \frac{1.5}{r - 0.02} - 10 \]

Setting NPV_B = 0 and solving for r

\[ \frac{1.5}{r - 0.02} = 10 \Rightarrow r - 0.02 = 0.15 \Rightarrow r = 17\% \text{ So, IRR}_B = 17\% \]

Based on the IRR you always pick project A

b. Substituting r = 0.07 into the NPV formulas derived in part (a) gives

NPV_A = $18.5714 million

NPV_B = $20 million

So the NPV says take B

c. Here is a plot of NPV of both projects as a function of the discount rate. The NPV rule selects A (and so agrees with the IRR rule) for all discount rates to the right of the point where the curves cross.

So the IRR rule will give the correct answer for discount rates greater than 8%
6-17.  
Timeline:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>-10</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>-10</td>
<td>1.5</td>
<td>1.5(1.02)</td>
</tr>
</tbody>
</table>

To calculate the incremental IRR subtract A from B:

\[
0 \quad 1.5 - 2 \quad 1.5(1.02) - 2 \quad 1.5(1.02)^2 - 2 \quad \cdots
\]

\[
\text{NPV} = \frac{1.5}{r - 0.02} - \frac{2}{r} = 0
\]

\[
\frac{2}{r} = \frac{1.5}{r - 0.02} \quad \frac{r}{r - 0.02} \quad \frac{2}{1.5}
\]

\[
1.5r = 2r - 0.04
\]

\[
0.5r = 0.04
\]

\[
r = 0.08
\]

So the incremental IRR is 8%. This rate is above the cost of capital so we should take B.

6-18.  
Timeline:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Playhouse</td>
<td>-30</td>
<td>15</td>
</tr>
<tr>
<td>Fort</td>
<td>-80</td>
<td>39</td>
</tr>
</tbody>
</table>

Subtract the Playhouse cash flows from the Fort:

\[
-54 \quad 24 \quad 32
\]

\[
\text{NPV} = -50 + \frac{24}{1 + r} + \frac{32}{(1 + r)^2}
\]

Solving for r:

\[
r = \frac{-2(50) + 24 + \sqrt{24^2 + 4(50)(32)}}{2(50)}
\]

\[
r = 7.522\%
\]

Since the incremental IRR of 7.522% is less than the cost of capital of 8% you should take the Playhouse.
6-19. Project NPV Profitability Index

<table>
<thead>
<tr>
<th>Project</th>
<th>NPV</th>
<th>Profitability Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parkside Acres</td>
<td>91,177</td>
<td>.18</td>
</tr>
<tr>
<td>Real Property Estates</td>
<td>120,523</td>
<td>.15</td>
</tr>
<tr>
<td>Lost Lake Properties</td>
<td>40,392</td>
<td>.06</td>
</tr>
<tr>
<td>Overlook</td>
<td>80,131</td>
<td>.53</td>
</tr>
</tbody>
</table>

They should select Overlook and Parkside Acres.

6-20. Project PI NPV/Headcount

<table>
<thead>
<tr>
<th>Project</th>
<th>PI</th>
<th>NPV/Headcount</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>.84</td>
<td>5.1</td>
</tr>
<tr>
<td>II</td>
<td>.75</td>
<td>3.8</td>
</tr>
<tr>
<td>III</td>
<td>.73</td>
<td>5.5</td>
</tr>
<tr>
<td>IV</td>
<td>1.61</td>
<td>8.3</td>
</tr>
<tr>
<td>V</td>
<td>1.51</td>
<td>7.5</td>
</tr>
</tbody>
</table>

a. It should select projects IV, V, I, and II

b. It should select IV and V.