Chapter 5  
Interest Rates

5-1.  
   a. Since 6 months is \( \frac{6}{24} = \frac{1}{4} \) of 2 years, using our rule \( (1 + 0.2)^{\frac{1}{4}} = 1.0466 \)  
      So the equivalent 6 month rate is 4.66%
   
   b. Since one year is half of 2 years \( (1.2)^{\frac{1}{2}} = 1.0954 \)  
      So the equivalent 1 year rate is 9.54%
   
   c. Since one month is \( \frac{1}{24} \) of 2 years, using our rule \( (1 + 0.2)^{\frac{1}{24}} = 1.00763 \)  
      So the equivalent 1 month rate is 0.763%

5-2.  
   If you deposit $1 into a bank account that pays 5% per year for 3 years you will have \( (1.05)^3 = 1.15763 \) after 3 years
   
   a. If the account pays \( 2 \frac{1}{2} \% \) per 6 months then you will have \( (1.025)^6 = 1.15969 \) after 3 years, so you prefer \( 2 \frac{1}{2} \% \) every 6 months
   
   b. If the account pays \( 7 \frac{1}{2} \% \) per 18 months then you will have \( (1.075)^2 = 1.15563 \) after 3 years, so you prefer 5% per year
   
   c. If the account pays \( \frac{1}{2} \% \) per month then you will have \( (1.005)^{36} = 1.19668 \) after 3 years, so you prefer \( \frac{1}{2} \% \) every month

5-3.  
   Timeline:
   
   \[
   \begin{array}{cccccc}
   0 & 7 & 14 & & 42 \\
   0 & 1 & 2 & & 6 \\
   \hline
   \end{array}
   \]

   \( (1.06)^7 = 1.50363 \)

   So the equivalent discount rate is 50.363%.

   Using the annuity formula

   \[
   PV = \frac{70,000}{0.50363} \left(1 - \frac{1}{(1.50363)^6} \right) = \$126,964
   \]
5-4. For a $1 invested in an account with 10% APR with monthly compounding you will have

\[
\left(1 + \frac{0.1}{12}\right)^{12} = \$1.10471
\]

So the EAR is 10.471%

For a $1 invested in an account with 10% APR with annual compounding you will have

\[
(1 + 0.1) = \$1.10
\]

So the EAR is 10%

For a $1 invested in an account with 9% APR with daily compounding you will have

\[
\left(1 + \frac{0.09}{365}\right)^{365} = 1.09416
\]

So the EAR is 9.416%

5-5. Using the formula for converting from an EAR to an APR quote

\[
\left(1 + \frac{\text{APR}}{k}\right)^k = 1.05
\]

Solving for the APR

\[
\text{APR} = \left(\frac{1.05}{k} - 1\right)^k
\]

With annual payments \( k = 1 \), so \( \text{APR} = 5\% \)

With semiannual payments \( k = 2 \), so \( \text{APR} = 4.939\% \)

With monthly payments \( k = 12 \), so \( \text{APR} = 4.889\% \)

5-6. Using the PV of an annuity formula with \( N = 10 \) payments and \( C = \$100 \) with \( r = 4.067\% \) per 6 month interval, since there is an 8% APR with monthly compounding: 8% / 12 = 0.6667% per month, or \( (1.006667)^6 - 1 = 4.067\% \) per 6 months.

\[
\text{PV} = 100 \times \frac{1}{0.04067} \left(1 - \frac{1}{1.04067^{10}}\right) = \$808.39
\]
5-7. Timeline:

\[
\begin{align*}
0 & \quad \frac{1}{2} \quad 1 \quad 2 \quad 4 \\
0 & \quad 1 \quad 2 \quad \ldots \ldots \quad 8 \\
10,000 & \quad 10,000 \quad \ldots \ldots \quad 10,000
\end{align*}
\]

4% APR (semiannual) implies a semiannual discount rate of \( \frac{4\%}{2} = 2\% \)

So,

\[
PV = \frac{10,000}{0.02} \left( 1 - \frac{1}{(1.02)^8} \right)
\]

\[
= 73,254.81
\]

5-8. Using the formula for computing the discount rate from an APR quote:

Discount Rate = \( \frac{5}{12} = 0.41667\% \)

5-9. Timeline:

\[
\begin{align*}
0 & \quad 1 \quad 2 \quad 3 \quad 4 \quad \ldots \ldots \quad 60 \\
-8,000 & \quad C \quad C \quad C \quad C \quad \ldots \ldots \quad C
\end{align*}
\]

5.99 APR monthly implies a discount rate of \( \frac{5.99}{12} = 0.499167\% \)

Using the formula for computing a loan payment

\[
C = \frac{1}{0.00499167} \left( \frac{1}{1 - \left( \frac{1}{1.00499167} \right)^{60}} \right) = 154.63
\]
5-10. Timeline:

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & \ldots & 360 \\
-150,000 & C & C & C & C & C & C \\
\end{array}
\]

\[
(1 + 0.05375)^{\frac{1}{12}} = 1.0043725
\]

So 5 3/4% EAR implies a discount rate of 0.43725%

Using the formula for computing a loan payment

\[
C = \frac{150,000}{0.0043725 \left(1 + \frac{1}{(1.0043725)^{360}}\right)} = \$828.02
\]

5-11. Timeline:

\[
\begin{array}{cccccc}
56 & 57 & 58 & \ldots & 360 \\
0 & 1 & 2 & \ldots & 304 \\
2,356 & 2,356 & \ldots & 2,356 \\
\end{array}
\]

To find out what is owed compute the PV of the remaining payments using the loan interest rate to compute the discount rate:

\[
\text{Discount Rate} = \frac{6.375}{12} = 0.53125\%
\]

\[
\text{PV} = \frac{2,356}{0.0053125 \left(1 - \frac{1}{(1.0053125)^{304}}\right)} = \$354,900
\]

5-12. First we need to compute the original loan payment

Timeline #1:

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & \ldots & 360 \\
-800,000 & C & C & C & C & C \\
\end{array}
\]

5 1/4% APR (monthly) implies a discount rate of \(\frac{5.25}{12} = 0.4375\%\)
Using the formula for a loan payment

\[ C = \frac{800,000 \times 0.004375}{1 - \frac{1}{(1.004375)^{360}}} = \$4,417.63 \]

Now we can compute the PV of continuing to make these payments

The timeline is

Timeline #2:

<table>
<thead>
<tr>
<th></th>
<th>222</th>
<th>223</th>
<th>224</th>
<th>225</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>138</td>
</tr>
<tr>
<td>PV</td>
<td>4,417.63</td>
<td>4,417.63</td>
<td>4,417.63</td>
<td>4,417.63</td>
<td>4,417.63</td>
</tr>
</tbody>
</table>

Using the formula for the PV of an annuity

\[ PV = \frac{4,417.63}{0.004375} \left(1 - \frac{1}{(1.004375)^{138}}\right) = \$456,931.41 \]

So, you would keep $1,000,000 - $456,931 = $543,069.

5-13.

a. APR of 6% = 0.5% per month. Payment = \( \frac{500,000}{0.005} \left(1 - \frac{1}{1.005^{360}}\right) = \$2997.75 \).

Total annual payments = 2997.75 \times 12 = \$35,973.

Loan balance at the end of 1 year = \(2997.75 \times 0.005 \left(1 - \frac{1}{1.005^{138}}\right) = \$493,860 \).

Therefore, 500,000 – 493,860 = \$6140 in principal repaid in first year, and 35,973 – 6140 = \$29833 in interest paid in first year.

b. Loan balance in 19 years (or 360 – 19\times12 = 132 remaining pmts) is

\[2997.75 \times 0.005 \left(1 - \frac{1}{1.005^{132}}\right) = \$289,162.\]

Loan balance in 20 years = \(2997.75 \times 0.005 \left(1 - \frac{1}{1.005^{132}}\right) = \$270,018.\)

Therefore, 289,162 – 270,018 = \$19,144 in principal repaid, and 35,973 – 19,144 = \$16,829 in interest repaid.
5-14. We begin with the timeline of our required payments

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>47</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-500</td>
<td></td>
<td>-500</td>
<td></td>
</tr>
</tbody>
</table>

(1) Let’s compute our remaining balance on the student loan. As we pointed out earlier, the remaining balance equals the present value of the remaining payments. The loan interest rate is 9% APR, or $9\% / 12 = 0.75\%$ per month, so the present value of the payments is

$$PV = \frac{500}{0.0075} \left( 1 - \frac{1}{1.0075^{48}} \right) = 20,092.39$$

Using the annuity spreadsheet to compute the present value, we get the same number:

<table>
<thead>
<tr>
<th>N</th>
<th>I</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>0.75%</td>
<td>20,092.39</td>
<td>-500</td>
<td>0</td>
</tr>
</tbody>
</table>

Thus, your remaining balance is $20,092.39.

If you prepay an extra $100 today, your will lower your remaining balance to $20,092.39 – 100 = $19,992.39. Though your balance is reduced, your required monthly payment does not change. Instead, you will pay off the loan faster; that is, it will reduce the payments you need to make at the very end of the loan. How much smaller will the final payment be? With the extra payment, the timeline changes:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>47</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>19,992.39</td>
<td></td>
<td>-500</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

That is, we will pay off by paying $500 per month for 47 months, and some smaller amount, $500 – X, in the last month. To solve for X, recall that the PV of the remaining cash flows equals the outstanding balance when the loan interest rate is used as the discount rate:

$$19,992.39 = \frac{500}{0.0075} \left( 1 - \frac{1}{(1 + 0.0075)^{48}} \right) - \frac{X}{1.0075^{48}}$$

Solving for X gives

$$19,992.39 = 20,092.39 - \frac{X}{1.0075^{48}}$$

$$X = 143.14$$

So the final payment will be lower by $143.14.
You can also use the annuity spreadsheet to determine this solution. If you prepay $100 today, and make payments of $500 for 48 months, then your final balance at the end will be a credit of $143.14:

<table>
<thead>
<tr>
<th>N</th>
<th>I</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>0.75%</td>
<td>19,992.39</td>
<td>-500</td>
<td>143.14</td>
</tr>
</tbody>
</table>

The extra payment effectively lets us exchange $100 today for $143.14 in four years. We claimed that the return on this investment should be the loan interest rate. Let’s see if this is the case:

\[
100 \times (1.0075)^{48} = 143.14, \text{ so it is.}
\]

(2) You earn a 9% APR (the rate on the loan).

5-15. The timeline in this case is:

\[
\begin{array}{cccc}
0 & 1 & 2 & N \\
20,092.39 & -750 & -750 & -750 \\
\end{array}
\]

and we want to determine the number of monthly payments \(N\) that we will need to make. That is, we need to determine what length annuity with a monthly payment of $750 has the same present value as the loan balance, using the loan interest rate as the discount rate. As we did in Chapter 4, we set the outstanding balance equal to the present value of the loan payments and solve for \(N\):

\[
\frac{750}{0.0075} \left(1 - \frac{1}{1.0075^N}\right) = 20,092.39
\]

\[
\left(1 - \frac{1}{1.0075^N}\right) = \frac{20,092.39 \times 0.0075}{750} = 0.200924
\]

\[
\frac{1}{1.0075^N} = 1 - 0.200924 = 0.799076
\]

\[
1.0075^N = 1.25145
\]

\[
N = \frac{\log(1.25145)}{\log(1.0075)} = 30.02
\]

We can also use the annuity spreadsheet to solve for \(N\):

<table>
<thead>
<tr>
<th>N</th>
<th>I</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.02</td>
<td>0.75%</td>
<td>20,092.39</td>
<td>-750</td>
<td>0</td>
</tr>
</tbody>
</table>
So, by prepaying the loan, we will pay off the loan in about 30 months or 2 ½ years, rather than the four years originally scheduled. Because $N$ of 30.02 is larger than 30, we could either increase the 30th payment by a small amount or make a very small 31st payment. We can use the annuity spreadsheet to determine the remaining balance after 30 payments:

<table>
<thead>
<tr>
<th>N</th>
<th>I</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.75%</td>
<td>20,092.39</td>
<td>-750</td>
<td>-13.86</td>
</tr>
</tbody>
</table>

If we make a final payment of $750.00 + $13.86 = $763.86, the loan will be paid off in 30 months.

5-16. From the solution to problem 5.10 the monthly payment on the mortgage is $828.02. So if we make $414.01 every 2 weeks the timeline is

Timeline:

<table>
<thead>
<tr>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>....</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>414.01</td>
<td>414.01</td>
<td>414.01</td>
<td>414.01</td>
<td>414.01</td>
<td></td>
</tr>
</tbody>
</table>

Now since there are 26 weeks in a year

\[ (1.05375)^{1/26} = 1.002016 \]

So, the discount rate is 0.2016%.

To compute $N$ we set the PV of the loan payments equal to the outstanding balance

\[ 150,000 = \frac{441.01}{0.002016} \left( 1 - \frac{1}{(1.002016)^N} \right) \]

And Solve for $N$

\[ 1 - \left( \frac{1}{1.002016} \right)^N = \frac{150,000 \times 0.002016}{441.01} = 0.7303 \]

\[ \left( \frac{1}{1.002016} \right)^N = 0.2697 \]

\[ N = \frac{\log(0.2697)}{\log \left( \frac{1}{1.002016} \right)} = 650.79 \]

So it will take 651 payments to pay off the mortgage. Since the payments occur every two weeks this will take $651 \times 2 = 1302$ weeks or approximately 25 years. (It is shorter because there are approximately 2 extra payments every year.)
5-17. The principle balance does not matter, so just pick 100,000. Begin by computing the monthly payment. The discount rate is \( \frac{12}{12} = 1\% \)

Timeline #1:

\[
\begin{array}{cccccc}
0 & 1 & 2 & \cdots & 360 \\
100,000 & -C & -C & \cdots & -C \\
\end{array}
\]

Using the formula for the loan payment

\[
C = \frac{100,000 \times 0.01}{\left(1 - \frac{1}{1.01^{360}}\right)} = \$1,028.61
\]

Next we write out the cash flows with the extra payment:

Timeline #2:

\[
\begin{array}{cccccccc}
0 & 1 & 6 & 7 & 18 & 19 & \cdots & N \\
100,000 & -1028.61 & -1028.61 & -1028.61 & -1028.61 & -1028.61 & \cdots & -1028.61 \\
\end{array}
\]

The cash flow consists of 2 annuities.

i. The original payments. The PV of these payments is

\[
PV_{org} = \frac{1,028.61}{0.01} \left(1 - \frac{1}{1.01^{N}}\right)
\]

ii. The extra payment every Christmas. There are \( \frac{N}{12} \) such payments. For the moment we will not worry about the possibility that \( \frac{N}{12} \) is not a whole number. Since the time period between payments is 1 year, we first have to compute the discount rate

\[
\left(1.1\right)^{12} = 1.12683
\]

So the discount rate is 12.683%.

Now the present value of the extra payments in month 6 consist of the remaining \( \frac{N}{12} - 1 \) payments and the payment in month 6. So the PV is

\[
PV_6 = \frac{1,028.61}{0.12683} \left(1 - \frac{1}{\left(1.12683\right)^{\frac{N}{12} - 1}}\right) + 1,028.61
\]
To get the value today we must discount these cash flows to month zero. Recall that the monthly discount rate is 1%. So the value today of the extra payment is:

\[
PV_{\text{extra}} = \frac{PV_6}{(1.01)^6} = \frac{1,028.61}{0.12683(1.01)^6} \left( 1 - \frac{1}{(1.12683)^{N-1}} \right) + \frac{1,028.61}{(1.01)^6}
\]

To find N we need to set the NPV of the cash flows equal to zero, and solve for N:

\[
NPV = 0 = 100,000 - PV_{\text{org}} - PV_{\text{extra}}
\]

\[
100,000 = \frac{1,028.61}{0.01} \left( 1 - \left( \frac{1}{1.01} \right)^N \right) + \frac{1,028.61}{0.12683(1.01)^6} \left( 1 - \frac{1}{(1.12683)^{N-1}} \right) + \frac{1,028.61}{(1.01)^6}
\]

The only way to find N is to iterate (guess). The answer is \( N = 228.53 = 19.04 \) years.

So, after exactly 19 years the PV of the payments is

\[
PV = \frac{1,028.61}{0.01} \left( 1 - \left( \frac{1}{1.01} \right)^{228} \right) + \frac{1,028.61}{0.12683(1.01)^6} \left( 1 - \frac{1}{(1.12683)^{18}} \right) + \frac{1,028.61}{(1.01)^6} = 99,939.16
\]

Since you initially borrowed $100,000 the PV of what you still owe at the end of 19 years is $100,000 – $99,939.16 = $60.84. The future value of this in 19 years and one month is

\[
60.84 \times (1.01)^{229} = 594.02
\]

So, you will have a partial payment of $594.02 in the first month of the 19th year. So the mortgage will take about 19 years to pay off this way which is close to \( \frac{2}{3} \) of its life of 30 years. So your friend is right.

5-18. You can use any money that you don’t spend on the car to pay down your credit card debt. Paying down the loan is equivalent to an investment earning the loan rate of 15% APR. Thus, your opportunity cost of capital is 15% APR (monthly) and so the discount rate is 15 / 12 = 1.25% per month. Computing the present value of option (ii) at this discount rate, we find

\[
PV(ii) = -5000 + (-500) \times \frac{1}{0.0125} \left( 1 - \frac{1}{1.0125^{30}} \right) = -5000 - 12,444 = -$17,444
\]

You are better off taking the loan from the dealer and using any extra money to pay down your credit card debt.
5-19.  

a. First we calculate the outstanding balance of the mortgage. The timeline is

Timeline #1:

<table>
<thead>
<tr>
<th></th>
<th>60</th>
<th>61</th>
<th>62</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td></td>
<td>300</td>
</tr>
<tr>
<td></td>
<td>1,402</td>
<td>1,402</td>
<td></td>
<td>1,402</td>
</tr>
</tbody>
</table>

To determine the outstanding balance we discount at the original rate, i.e., \( \frac{10}{12} = 0.8333\% \)

\[
PV = \frac{1402}{0.008333} \left(1 - \frac{1}{(1.008333)^{300}}\right) = $154,286.22
\]

Next we calculate the loan payment on the new mortgage

Timeline #2:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>360</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>154,286.22</td>
<td>–C</td>
<td>–C</td>
<td>–C</td>
</tr>
</tbody>
</table>

The discount rate on the new loan is the new loan rate: \( \frac{6.625}{12} = 0.5521\% \)

Using the formula for the loan payment:

\[
C = \frac{154,286.22 \times 0.005521}{1 - \left(\frac{1}{1.005521}\right)^{360}} = $987.93
\]

So, your payments drop by 

\(1,402 - 987.93 = $414.07\) .

b. \(C = \frac{154,286.22 \times 0.005521}{1 - \left(\frac{1}{1.005521}\right)^{300}} = $1,053.85\)
c. \[ PV = \frac{1402}{0.005521} \left(1 - \frac{1}{(1.005521)^N}\right) = 154,286.22 \Rightarrow N = 170 \text{ months} \]

d. \[ PV = \frac{1402}{0.005521} \left(1 - \frac{1}{(1.005521)^{300}}\right) = 205,255 \]

\[ \Rightarrow \text{you can keep } 205,259 - 154,286 = 50,969 \]

5-20. The discount rate on the original card is

\[ \frac{15}{12} = 1.25\% \]

Assuming that your current monthly payment is the interest that accrues, it equals:

\[ $25,000 \times \frac{0.15}{12} = $312.50 \]

Timeline:

\[ \begin{array}{cccc}
0 & 1 & 2 \\
\hline
312.50 & 312.50 \\
\end{array} \]

This is a perpetuity. So the amount you can borrow at the new interest rate is this cash flow discounted at the new discount rate. The new discount rate is \[ \frac{12}{12} = 1\% \]

So, \[ PV = \frac{312.50}{0.01} = $31,250 \]

So by switching credit cards you are able to spend an extra \[ 31,250 - 25,000 = $6,250 \]
You do not have to pay taxes on this amount of new borrowing, so this is your after-tax benefit of switching cards

5-21. \[ r_t = \frac{r - i}{1 + i} = \frac{7.85\% - 12.3\%}{1.123} = -3.96\% \]

The purchasing power of your savings declined by 3.96% over the year.

5-22. \[ 1 + r_t = \frac{1 + r}{1 + i} \text{ implies } 1 + r = (1 + r_t)(1 + i) = (1.03)(1.05) = 1.0815. \]

Therefore, a nominal rate of 8.15% is required.
5-23. By holding cash, an investor earns a nominal interest rate of 0%. Since an investor can always earn at least 0%, the nominal interest rate cannot be negative. The real interest rate can be negative, however. It is negative whenever the rate of inflation exceeds the nominal interest rate.

5-24.  
   a. \( \text{NPV} = -100,000 + \frac{150,000}{1.055} = $17,529. \)
   b. \( \text{NPV} = -100,000 + \frac{150,000}{1.105} = -$6862 \)
   c. The answer is the IRR of the investment. \( \text{IRR} = (\frac{150,000}{100,000})^{1/5} - 1 = 8.45\% \)

5-25.  
   a. Timeline:

   ![Timeline](image)

   Since the opportunity cost of capital is different for investments of different maturities, we must use the cost of capital associated with each cash flow as the discount rate for that cash flow:

   \[
   \text{PV} = \frac{1,000}{(1.0241)^2} + \frac{2,000}{(1.0332)^3} = $2,652.15
   \]

   b. Timeline:

   ![Timeline](image)

   Since the opportunity cost of capital is different for investments of different maturities, we must use the cost of capital associated with each cash flow as the discount rate for that cash flow. Unfortunately we do not have a rate for a 4 year cash flow, so we linearly interpolate:

   \[
   r_t = \frac{1}{2} (2.74) + \frac{1}{2} (3.32) = 3.03
   \]

   \[
   \text{PV} = \frac{500}{1.0199} + \frac{500}{(1.0241)^2} + \frac{500}{(1.0274)^3} + \frac{500}{(1.0303)^4} + \frac{500}{(1.0332)^5} = $2,296.43
   \]
c. Timeline:

Since the opportunity cost of capital is different for investments of different maturities, we must use the cost of capital associated with each cash flow as the discount rate for that cash flow. Unfortunately we do not have a rate for a number of years, so we linearly interpolate:

\[
r_i = \frac{1}{2}(2.74) + \frac{1}{2}(3.32) = 3.03
\]
\[
r_i = \frac{1}{2}(3.32) + \frac{1}{2}(3.76) = 3.54
\]
\[
r_i = \frac{2}{3}(3.76) + \frac{1}{3}(4.13) = 3.883
\]
\[
r_i = \frac{1}{3}(3.76) + \frac{2}{3}(4.13) = 4.0067
\]
\[
r_{11} = \frac{9}{10}(4.13) + \frac{1}{10}(4.93) = 4.21
\]
\[
r_{12} = \frac{8}{10}(4.13) + \frac{2}{10}(4.93) = 4.29
\]
\[
r_{13} = 4.37
\]
\[
r_{14} = 4.45
\]
\[
r_{15} = 4.53
\]
\[
r_{16} = 4.61
\]
\[
r_{17} = 4.64
\]
\[
r_{18} = 4.77
\]
\[
r_{19} = 4.85
\]

\[
PV = \frac{2,300}{1 + r_i} + \frac{2,300}{(1 + r_i)^2} + \frac{2,300}{(1 + r_i)^3} + \cdots + \frac{2,300}{(1 + r_{20})^{20}}
\]
\[
= \frac{2,300}{1.0199} + \frac{2,300}{1.0241} + \frac{2,300}{1.0274} + \cdots + \frac{2,300}{(1.0493)^{20}}
\]
\[
= 30,636.56
\]
5-26. PV = 100 / 1.0199 + 100 / 1.02412 + 100 / 1.02743 = $285.61.

To determine the single discount rate that would compute the value correctly, we solve for the following for r:

\[ PV = 100/(1 + r) + 100 / (1 + r)^2 + 100/(1 + r)^3 = $285.61. \]

This is just an IRR calculation. Using trial and error or the annuity calculator, \( r = 2.50\% \). Note that this rate is between the 1, 2, and 3-yr rates given.

5-27. The yield curve is increasing. This is often a sign that investors expect interest rates to rise in the future.

5-28.  
   a. The 1-year interest rate is 6%. If rates fall next year to 5%, then if you reinvest at this rate over two years you would earn \((1.06)(1.05) = 1.113\) per dollar invested. This amount corresponds to an EAR of \((1.113)^{1/2} − 1 = 5.50\%\) per year for two years. Thus, the two-year rate that is consistent with these expectations is 5.50%.

   b. We can apply the same logic for future years:

<table>
<thead>
<tr>
<th>Year</th>
<th>Future Interest Rates</th>
<th>FV from reinvesting</th>
<th>EAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6%</td>
<td>1.0600</td>
<td>6.00%</td>
</tr>
<tr>
<td>2</td>
<td>5%</td>
<td>1.1130</td>
<td>5.50%</td>
</tr>
<tr>
<td>3</td>
<td>2%</td>
<td>1.1353</td>
<td>4.32%</td>
</tr>
<tr>
<td>4</td>
<td>3%</td>
<td>1.1693</td>
<td>3.99%</td>
</tr>
<tr>
<td>5</td>
<td>4%</td>
<td>1.2161</td>
<td>3.99%</td>
</tr>
<tr>
<td>6</td>
<td>5%</td>
<td>1.2769</td>
<td>4.16%</td>
</tr>
<tr>
<td>7</td>
<td>6%</td>
<td>1.3535</td>
<td>4.42%</td>
</tr>
<tr>
<td>8</td>
<td>6%</td>
<td>1.4347</td>
<td>4.62%</td>
</tr>
<tr>
<td>9</td>
<td>6%</td>
<td>1.5208</td>
<td>4.77%</td>
</tr>
<tr>
<td>10</td>
<td>6%</td>
<td>1.6121</td>
<td>4.89%</td>
</tr>
</tbody>
</table>

   c. We can plot the yield curve using the EAR’s in (b), note that the 10-yr rate is below the 1-yr rate (yield curve is inverted).

5-29. We can use the interest rates each company must pay on a 5-year loan as the discount rate.

   PV for GM = 700 / 1.0822^5 = $471.59 < $500 today, so take the money now.

   PV for JP Morgan = 700 / 1.0544^5 = $537.12 > $500 today, so take the promise.

5-30. After-tax rate = 4%(1 − .30) = 2.8%, which is less than your tax-free investment with pays 3%.

5-31. After-tax cost of home equity loan is 8%(1 − .25) = 6%, which is cheaper than the dealer’s loan (for which interest is not tax-deductible). Thus, the home equity loan is cheaper. (Note that this could also be done in terms of EARs.)
5-32. Using the formula to convert an APR to an EAR:

\[
\left(1 + \frac{0.06}{12}\right)^{12} = 1.06168
\]

So the home equity loan as an EAR 6.168%. Now since the rate on a tax deductible loan is a before tax rate, we must convert this to an after tax rate to compare it

\[6.168 \times (1 - 0.15) = 5.243\%
\]

Since the student loan has a larger after tax rate, you are better off using the home equity loan.

5-33.

a. The regular savings account pays 5.5% EAR, or \(5.5\% (1 - .35) = 3.575\% \) after-tax. The money-market account pays \((1 + 5.25\% / 365)365 - 1 = 5.39\%\), or \(5.39\% (1 - .35) = 3.50\% \) after-tax. Therefore, the regular savings account pays a higher rate.

b. Your friend should pay off the credit card loans and the car loan, since they have after-tax costs of 14.9\% APR and 4.8\% APR respectively, which exceed the rate earned on savings. The home equity loan should not be repaid, as its EAR = \((1 + 5\% / 12)12 - 1 = 5.12\%\), for an after-tax rate of only \(5.12\% (1 - .35) = 3.33\%\) which is below the rate earned on savings.

5-34. 8\% is the appropriate cost of capital for a new risk-free investment, since you could earn 8\% without risk by paying off your existing loan and avoiding interest charges.