Chapter 4
The Time Value of Money

4-1.

From the bank’s perspective the timeline is the same except all the signs are reversed.

4-2.
From the bank’s perspective the timeline would be identical except with opposite signs.

4-3.

a. Timeline:

\[
\begin{array}{cccccc}
0 & 1 & 2 & \cdots & 5 \\
2000 & & & & & FV = ?
\end{array}
\]

\[FV_5 = 2,000 \times 1.05^5 = 2,552.56\]

b. Timeline:

\[
\begin{array}{cccccc}
0 & 1 & 2 & \cdots & 10 \\
2000 & & & & & FV = ?
\end{array}
\]

\[FV_{10} = 2,000 \times 1.05^{10} = 3,257.79\]
c. Timeline:

```
 0 1 2 3 4 5
```

$$2000 \hfill FV=?$$

$$FV_2 = 2000 \times 1.1^5 = 3221.02$$

d. Because in the last 5 years you get interest on the interest earned in the first 5 years as well as interest on the original $2,000.

4-4.

a. Timeline:

```
 0 1 2 3 4 5
```

$$PV=? \hfill 10,000$$

$$PV = \frac{10,000}{1.04^{12}} = 6245.97$$

b. Timeline:

```
 0 1 2 3 4 5
```

$$PV=? \hfill 10,000$$

$$PV = \frac{10,000}{1.08^{20}} = 2145.48$$

c. Timeline:

```
 0 1 2 3 4 5 6
```

$$PV=? \hfill 10,000$$

$$PV = \frac{10,000}{1.02^6} = 8879.71$$
4-5. Timeline:

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PV = ?

$PV = \frac{10,000}{1 + 0.07^{10}} = 5,083.49$

So the 10,000 in 10 years is preferable because it is worth more.

4-6. Timeline:

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PV = ?

$PV = \frac{100,000}{1 + 0.03^{10}} = 74,409.39$

4-7. Timeline: Same for all parts

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PV = ?

a. $PV = \frac{350,000}{1 + 0.05^5} = 350,000$

So you should take the 350,000

b. $PV = \frac{350,000}{1 + 0.08^5} = 238,204$

You should take the 250,000.

c. $PV = \frac{350,000}{1 + 0.2^5} = 140,657$

You should take the 250,000.
4-8.  

a. Timeline:

\[
\begin{align*}
\text{Timeline:} & \\
18 & 19 & 20 & 21 & 25 \\
0 & 1 & 2 & 3 & 7 \\
\end{align*}
\]

\[
3,996 \\
FV=?
\]

\[
FV = 3,996(1.08)^7 \\
= 6,848.44
\]

b. Timeline:

\[
\begin{align*}
\text{Timeline:} & \\
18 & 19 & 20 & 21 & 65 \\
0 & 1 & 2 & 3 & 47 \\
\end{align*}
\]

\[
3,996 \\
FV ?
\]

\[
FV = 3,996(1.08)^{47} = 148,779
\]

c. Timeline:

\[
\begin{align*}
\text{Timeline:} & \\
0 & 1 & 2 & 3 & 4 & 18 \\
\end{align*}
\]

\[
PV=? \\
3,996
\]

\[
PV = \frac{3,996}{1.08^{18}} = 1,000
\]

4-9.  

a. Timeline:

\[
\begin{align*}
\text{Timeline:} & \\
0 & 1 & 2 & 3 \\
10,000 & 20,000 & 30,000 \\
\end{align*}
\]

\[
PV = \frac{10,000}{1.035} + \frac{20,000}{1.035^2} + \frac{30,000}{1.035^3} \\
= 9,662 + 18,670 + 27,058 = 55,390
\]
b. Timeline:

\[
\begin{array}{c|c|c|c}
0 & 1 & 2 & 3 \\
10,000 & 20,000 & 30,000 \\
\end{array}
\]

\[FV = 55,390 \times 1.035^3 = 61,412\]

4-10. Timeline:

\[
\begin{array}{c|c|c|c}
0 & 1 & 2 & 3 \\
1,000 & 1,000 & 1,000 \\
\end{array}
\]

First, calculate the present value of the cash flows:

\[
PV = \frac{1,000}{1.05} + \frac{1,000}{1.05^2} + \frac{1,000}{1.05^3} = 952 + 907 + 864 = 2,723
\]

Once you know the present value of the cash flows, compute the future value (of this present value) at date 3.

\[FV_3 = 2,723 \times 1.05^3 = 3,152\]

4-11. Timeline:

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c}
0 & 1 & 2 & 3 & \cdots & 10 \\
-10,000 & 1,500 & 1,500 & 500 \\
- & - & - & - & - & - & - \\
10,000 & 1,020 & 1,020 & 1,020 \\
\end{array}
\]

a. \[
NPV = -10,000 + \frac{500}{1.06} + \frac{1,500}{1.06^2} + \frac{10,000}{1.06^{10}} = -10,000 + 471.70 + 1,334.99 + 5,583.95 = -2,609.36
\]

Since the NPV < 0, don’t take it.

b. \[
NPV = -10,000 + \frac{500}{1.02} + \frac{1,500}{1.02^2} + \frac{10,000}{1.02^{10}} = -10,000 + 490.20 + 1,441.75 + 8,203.48 = 135.43
\]

Since the NPV > 0, take it.
4-12. Timeline:

\[
\begin{align*}
\text{Year} & \quad 0 & 1 & 2 & 3 \\
\text{Cash Flow} & \quad -1,000 & 4,000 & -1,000 & 4,000 \\
\text{NPV} & \quad & 1.02 & & 1.02 & 1.02 & 1.02 \\
\end{align*}
\]

\[
NPV = -1,000 + \frac{4,000}{(1.02)} - \frac{1,000}{(1.02)^2} + \frac{4,000}{(1.02)^3}
\]

\[
= -1,000 + 3,921.57 - 961.17 + 3,769.29 = 5,729.69
\]

Yes, make the investment.

4-13. Timeline:

\[
\begin{align*}
\text{Year} & \quad 0 & 1 & 2 & 3 \\
\text{Cash Flow} & \quad -1,000 & 100 & 100 & 100 \\
\text{PV} & \quad & & & \\
\end{align*}
\]

To decide whether to build the machine you need to calculate the NPV. The cash flows the machine generates are a perpetuity, so by the PV of a perpetuity formula:

\[
PV = \frac{100}{0.095} = 1,052.63
\]

So the \( NPV = 1,052.63 - 1,000 = 52.63 \). He should build it.

4-14. Timeline:

\[
\begin{align*}
\text{Year} & \quad 0 & 1 & 2 & 3 \\
\text{Cash Flow} & \quad -1,000 & 100 & 100 & \\
\text{PV} & \quad & & & \\
\end{align*}
\]

To decide whether to build the machine you need to calculate the NPV: The cash flows the machine generates are a perpetuity with first payment at date 2. Computing the PV at date 1 gives

\[
PV_1 = \frac{100}{0.095} = 1,052.63
\]

So the value today is

\[
PV_0 = \frac{1,052.63}{1.095} = 961.31 \quad \text{So the } NPV = 961.31 - 1,000 = -38.69
\]

He should not build the machine.
4-15.  Timeline:

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
100 & 100 & 100 & \cdots \\
\end{array}
\]

a. The value of the bond is equal to the present value of the cash flows. By the perpetuity formula:

\[
P_V = \frac{100}{0.04} = £2,500
\]

b. The value would be the same, £2,500 after each payment.

4-16.  Timeline:

\[
\begin{array}{ccccc}
0 & 1 & 2 & 3 & 100 \\
1,000 & 1,000 & 1,000 & \cdots \\
1,000 & & & & \\
\end{array}
\]

The cash flows are a 100 year annuity, so by the annuity formula:

\[
P_V = \frac{1,000}{0.07} \left( \frac{1}{1.07^{100}} \right) = 14,269.25
\]

4-17.  Timeline:

\[
\begin{array}{cccc}
0 & 5 & 10 & 20 \\
0 & 1 & 2 & 3 \\
1,000,000 & 1,000,000 & 1,000,000 & \cdots \\
\end{array}
\]

First we need the 5-year interest rate. If the annual interest rate is 8% per year and you invest $1 for 5 years you will have, by the 2nd rule of time travel, \((1.08)^5 = 1.46932808\). So the 5 year interest rate is 46.93%. The cash flows are a perpetuity, so:

\[
P_V = \frac{1,000,000}{0.46932808} = 2,130,833
\]
4-18.

a. Timeline:

\[
\begin{array}{cccccc}
12 & 13 & 14 & 15 & 30 \\
0 & 1 & 2 & 3 & 18 \\
\hline
1,200 & 1,200 & 1,200 & 1,200 \\
\hline
\end{array}
\]

To pay off the mortgage you must repay the remaining balance. The remaining balance is equal to the present value of the remaining payments. The remaining payments are an 18 year annuity, so:

\[
PV = \frac{1,200}{0.06} \left(1 - \frac{1}{1.06^{18}}\right) = 12,993.12
\]

b. Timeline:

\[
\begin{array}{cccccc}
21 & 22 & 23 & 24 & 30 \\
0 & 1 & 2 & 3 & 10 \\
\hline
1,200 & 1,200 & 1,200 & 1,200 \\
\hline
\end{array}
\]

To pay off the mortgage you must repay the remaining balance. The remaining balance is equal to the present value of the remaining payments. The remaining payments are an 18 year annuity, so:

\[
PV = \frac{1,200}{0.06} \left(1 - \frac{1}{1.06^{18}}\right) = 8,832.10
\]

c. Timeline:

\[
\begin{array}{cccccc}
12 & 13 & 14 & 15 & 30 \\
0 & 1 & 2 & 3 & 18 \\
\hline
1,200 & 1,200 & 1,200 & 1,200 \\
\hline
\end{array}
\]

If you decide to pay off the mortgage immediately before the 12th payment, you will have to pay exactly what you paid in part (a) as well as the 12th payment itself:

\[
12,993.12 + 1,200 = 14,193.12
\]
4-19. Timeline:

We first calculate the present value of the deposits at date 0. The deposits are an 18 year annuity:

\[ PV = \frac{1,000}{0.03} \left( 1 - \frac{1}{1.03^{18}} \right) = 13,753.51 \]

Now, we calculate the future value of this amount:

\[ FV = 13,753.51(1.03)^{18} = 23,414.43 \]

4-20.

a. Timeline:

Using the formula for the PV of a growing perpetuity gives:

\[ PV = \frac{1,000}{0.12 - 0.08} = 25,000 \]

b. Timeline:

Using the formula for the PV of a growing perpetuity gives:

\[ PV = \frac{1,000(1.08)}{0.12 - 0.08} = 27,000 \]
4-21. Timeline:

We must value a growing perpetuity with a negative growth rate of -0.02:

\[ PV = \frac{1,000}{0.05 - (-0.02)} = 14,285.71 \]

4-22. Timeline:

This is a 17-year growing annuity. By the growing annuity formula we have

\[ PV = \frac{2,000,000}{0.1 - 0.05} \left(1 - \frac{1.05}{1.1}\right)^{17} = 21,861,455.80 \]

4-23. Timeline:

This problem consists of 2 parts: today’s tuition payment of $10,000 and a 12-year growing annuity with first payment of $10,000(1.05). However, we cannot use the growing annuity formula because in this case \( r = g \). We can just calculate the present values of the payments and add them up:

\[ PV_{GA} = \frac{10,000(1.05)}{1.05} + \frac{10,000(1.05)^2}{(1.05)^2} + \frac{10,000(1.05)^3}{(1.05)^3} + \cdots + \frac{10,000(1.05)^{12}}{(1.05)^{12}} \]
\[ = 10,000 + 10,000 + 10,000 + \cdots + 10,000 = 10,000 \times 12 \]
\[ = 120,000 \]

Adding the initial tuition payment gives:

\[ 120,000 + 10,000 = 130,000 \]
4-24. Timeline:

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & \cdots & 20 \\
\hline
5,000 & 5000(1.05) & 5000(1.05)^2 & & & & & 5000(1.05)^{19}
\end{array}
\]

This value is equal to the PV of a 20-year annuity with a first payment of $5,000. However we cannot use the
growing annuity formula because in this case \( r = g \). So instead we can just find the present values of the
payments and add them up:

\[
P_{VA} = \frac{5,000}{(1.05)} + \frac{5,000(1.05)}{(1.05)^2} + \frac{5,000(1.05)^2}{(1.05)^3} + \cdots + \frac{5,000(1.05)^{19}}{(1.05)^{20}}
\]

\[
= \frac{5,000}{1.05} + \frac{5,000}{1.05} + \frac{5,000}{1.05} + \cdots + \frac{5,000}{1.05} = 5,000 \times 20 = 95,238
\]

4-25. Timeline:

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
\hline
1(1.3) & (1.3)^2 & (1.3)^3 & (1.3)^4 & (1.3)^5 & (1.3)^5(1.02) & (1.3)^5(1.02)^2
\end{array}
\]

This problem consists of two parts:

1. A growing annuity for 5 years
2. A growing perpetuity after 5 years

First we find the PV of (1)

\[
P_{VA} = \frac{1.3}{0.08 - 0.3} \left[ 1 - \left( \frac{1.3}{1.08} \right)^5 \right] = $9.02\ million
\]

Now we calculate the PV of (2). The value at date 5 of the growing perpetuity is

\[
PV_5 = \frac{(1.3)^5(1.02)}{0.08 - 0.02} = $63.12\ million \Rightarrow PV_0 = \frac{63.12}{(1.08)^5} = $42.96\ million
\]

Adding the present value of (1) and (2) together gives the PV value of future earnings:

\[
$9.02 + $42.96 = $51.98\ million
\]
4-26. Timeline:

\[
\begin{array}{cccc}
0 & 1 & 2 & 3 \\
-1,000 & C & C & C \\
\end{array}
\]

\[P = \frac{C}{r} \Rightarrow C = P \times r = 1,000 \times 0.05 = $50\]

4-27. Timeline: (From the perspective of the bank)

\[
\begin{array}{cccccccc}
0 & 1 & 2 & 3 & \ldots & 30 \\
-300,000 & C & C & C & C & C \\
\end{array}
\]

\[C = \frac{300,000}{\frac{1}{0.07} \left(1 - \frac{1}{1.07^{30}}\right)} = $24,176\]

4-28. Timeline:

\[
\begin{array}{cccccccc}
0 & 2 & 4 & 6 & \ldots & 20 \\
0 & 1 & 2 & 3 & \ldots & 10 \\
-50,000 & C & C & C & \ldots & C \\
\end{array}
\]

This cash flow stream is an annuity. First, calculate the 2-year interest rate: the 1-year rate is 4%, and $1 today will be worth \((1.04)^2 = 1.0816\) in 2 years, so the 2-year interest rate is 8.16%. Using the equation for an annuity payment:

\[C = \frac{50,000}{\frac{1}{0.0816} \left(1 - \frac{1}{(1.0816)^{10}}\right)} = $7,505.34\]
4-29. Timeline: (where X is the balloon payment.)

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & \ldots & 30 \\
-300,000 & 23,500 & 23,500 & 23,500 & \ldots & 23,500 + X \\
\end{array}
\]

The present value of the loan payments must be equal to the amount borrowed:

\[
300,000 = \frac{23,500}{0.07} \left( 1 - \frac{1}{1.07^{30}} \right) + \frac{X}{(1.07)^{30}}
\]

Solving for X:

\[
X = \left[ 300,000 - \frac{23,500}{0.07} \left( 1 - \frac{1}{1.07^{30}} \right) \right] (1.07)^{30} = \$63,848
\]

4-30. Timeline:

\[
\begin{array}{cccccc}
30 & 31 & 32 & 33 & 65 \\
0 & 1 & 2 & 3 & \ldots & 35 \\
C & C & C & C & \ldots & C \\
\end{array}
\]

FV = $2 million

The PV of the cash flows must equal the PV of $2 million in 35 years. The cash flows consist of a 35-year annuity, plus contribution today, so the PV is:

\[
PV = \frac{C}{0.05} \left( 1 - \frac{1}{(1.05)^{35}} \right) + C
\]

The PV of $2 million in 35 years is

\[
\frac{2,000,000}{(1.05)^{35}} = \$362,580.57
\]

Setting these equal gives:

\[
\frac{C}{0.05} \left( 1 - \frac{1}{(1.05)^{35}} \right) + C = 362,580.57
\]

\[
\Rightarrow C = \frac{362,580.57}{\frac{1}{0.05} \left( 1 - \frac{1}{(1.05)^{35}} \right) + 1} = \$20,868.91
\]
FV = 2 million

The PV of the cash flows must equal the PV of $2 million in 35 years. The cash flow consists of a 35 year growing annuity, plus the contribution today. So the PV is:

\[ PV = \frac{C(1.07)}{0.05 - 0.07} \left(1 - \frac{(1.07)^{35}}{1.05}\right) + C \]

The PV of $2 million in 35 years is:

\[ \frac{2,000,000}{(1.05)^{35}} = \$362,580.57 \]

Setting these equal gives:

\[ \frac{C(1.07)}{0.05 - 0.07} \left(1 - \frac{(1.07)^{35}}{1.05}\right) + C = 362,580.57 \]

Solving for C

\[ C = \frac{362,580.57}{\frac{1.07}{0.05 - 0.07} \left(1 - \frac{(1.07)^{35}}{1.05}\right) + 1} = \$7,102.11 \]

4-32. Timeline:

\[ \begin{array}{cccccc}
0 & 1 & \hspace{1cm} & \hspace{1cm} & \hspace{1cm} & 65 \\
\text{C} & \text{C}(1.07) & \text{C}(1.07)^2 & \text{C}(1.07)^3 & \ldots & \text{C}(1.07)^{35} \\
\end{array} \]

IRR is the r that solves:

\[ \frac{6,000}{1 + r} = \frac{5,000}{6,000} - 1 = 20\% \]
4-33. Timeline:

```
0  1  2  3  4
-32,500 10,000 10,000 10,000 10,000
```

The PV of the car payments is a 4-year annuity:

\[
PV = \frac{10,000}{r} \left[1 - \left(\frac{1}{1+r}\right)^4\right]
\]

Setting the NPV of the cash flow stream equal to zero and solving for \( r \) gives the IRR:

\[
NPV = 0 = -32,500 + \frac{10,000}{r} \left(1 - \left(\frac{1}{1+r}\right)^4\right) \Rightarrow \frac{10,000}{r} \left(1 - \left(\frac{1}{1+r}\right)^4\right) = 32,500
\]

To find \( r \) we either need to guess or use the annuity calculator. You can check and see that \( r = 8.85581\% \) solves this equation. So the IRR is 8.86%.

4-34. Timeline:

```
0  1  2  3
-1,000 100 100 100
```

The payments are a perpetuity, so

\[
PV = \frac{100}{r}
\]

Setting the NPV of the cash flow stream equal to zero and solving for \( r \) gives the IRR:

\[
NPV = 0 = \frac{100}{r} - 1,000 \Rightarrow r = \frac{100}{1,000} = 10\%
\]

So the IRR is 10%
The PV of the cash flows generated by storing the cheese is:

\[
PV = \frac{47.45}{(1 + r)^7} + \frac{32.85}{(1 + r)^{13}} + \frac{23.90}{(1 + r)^{22}}
\]

The IRR is the \( r \) that sets the NPV equal to zero:

\[
NPV = 0 = -79.50 + \frac{47.45}{(1 + r)^7} + \frac{32.85}{(1 + r)^{13}} + \frac{23.90}{(1 + r)^{22}}
\]

At this point we need to find one \( r \) that solves this equation by iteration. The IRR is 2.2% per month.

She breaks even when the NPV of the cash flows is zero. The value of \( N \) that solves this is:

\[
NPV = -200,000 + \frac{25,000}{0.05} \left(1 - \frac{1}{(1.05)^N}\right) = 0
\]

\[
\Rightarrow 1 - \frac{1}{(1.05)^N} = \frac{200,000 \times 0.05}{25,000} = 0.4
\]

\[
\frac{1}{(1.05)^N} = 0.6 \Rightarrow (1.05)^N = \frac{1}{0.6}
\]

\[
\log (1.05)^N = \log \left( \frac{1}{0.6} \right)
\]

\[
N \log (1.05) = -\log (0.6)
\]

\[
N = \frac{-\log (0.6)}{\log (1.05)} = 10.5
\]

So if she lives 10.5 or more years she comes out ahead.
4-37. Timeline:

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<th>\cdots</th>
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<tr>
<td>-10,000,000</td>
<td>1,000,000 - 50,000</td>
<td>1,000,000 - 50,000</td>
<td>\cdots</td>
<td>1,000,000 - 50,000</td>
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The plant will shut down when:

\[
1,000,000 - 50,000 \left(1.05\right)^{N-1} < 0
\]

\[
\left(1.05\right)^{N-1} > \frac{1,000,000}{50,000} = 20
\]

\[
(N-1) \log (1.05) = \log (20)
\]

\[
N = \frac{\log (20)}{\log (1.05)} + 1 = 62.4
\]

So the last year of production will be in year 62.

The cash flows consist of two pieces, the 62 year annuity of the $1,000,000 and the growing annuity.

The PV of the annuity is

\[
PV_A = \frac{1,000,000}{0.06} \left(1 - \frac{1}{(1.06)^{62}}\right) = 16,217,006
\]

The PV of the growing annuity is

\[
PV_{GA} = -\frac{50,000}{0.06 - 0.05} \left(1 - \frac{1.05}{1.06}^{62}\right) = -2,221,932
\]

So the PV of all the cash flows is

\[
PV = 16,217,006 - 2,221,932 = $13,995,074
\]

So the NPV = $13,995,074 - 10,000,000 = $3,995,074 and you should build it.
Timeline:

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The present value of the costs must equal the PV of the benefits. So begin by dividing the problem into two parts, the costs and the benefits.

**Costs:** The costs are the contributions, a 35-year annuity with the first payment in one year:

\[ PV_{\text{costs}} = \frac{C}{0.07} \left( 1 - \frac{1}{(1.07)^{35}} \right) \]

**Benefits:** The benefits are the payouts after retirement, a 35-year annuity paying $100,000 per year with the first payment 36 years from today. The value of this annuity in year 35 is:

\[ PV_{35} = \frac{100,000}{0.07} \left( 1 - \frac{1}{(1.07)^{35}} \right) \]

The value today is just the discounted value in 35 years:

\[ PV_{\text{benefits}} = \frac{PV_{35}}{(1.07)^{35}} = \frac{100,000}{0.07(1.07)^{35}} \left( 1 - \frac{1}{(1.07)^{35}} \right) = 121,272 \]

Since the PV of the costs must equal the PV of the benefits (or equivalently the NPV of the cash flow must be zero):

\[ 121,272 = \frac{C}{0.07} \left( 1 - \frac{1}{(1.07)^{35}} \right) \]

Solving for C gives:

\[ C = \frac{121,272 \times 0.07}{1 - \frac{1}{(1.07)^{35}}} = 9,366.29 \]
4-39. Timeline: (f = Fraction of your salary that you contribute)

<table>
<thead>
<tr>
<th>30</th>
<th>31</th>
<th>32</th>
<th>65</th>
<th>66</th>
<th>67</th>
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<td>2</td>
<td>35</td>
<td>36</td>
<td>37</td>
<td>70</td>
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<tr>
<td>75f</td>
<td>75(1.02)f</td>
<td>75(1.02)^34f</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

The present value of the costs must equal the PV of the benefits. So begin by dividing the problem into two parts, the costs and the benefits.

**Costs:** The costs are the contributions, a 35-year growing annuity with the first payment in one year. The PV of this is

\[
P_{\text{costs}} = \frac{75,000f}{0.07 - 0.02} \left(1 - \left(\frac{1.02}{1.07}\right)^{35}\right)
\]

**Benefits:** The benefits are the payouts after retirement, a 35-year annuity paying $100,000 per year with the first payment 36 years from today. The value of this annuity in year 35 is

\[
P_{35} = \frac{100,000}{0.07} \left(1 - \frac{1}{(1.07)^{35}}\right)
\]

The value today is just the discounted value in 35 years.

\[
P_{\text{benefits}} = \frac{P_{35}}{(1.07)^{35}} = \frac{100,000}{0.07(1.07)^{35}} \left(1 - \frac{1}{(1.07)^{35}}\right) = 121,272
\]

Since the PV of the costs must equal the PV of the benefits (or equivalently the NPV of the cash flows must be zero):

\[
121,272 = \frac{75,000f}{0.07 - 0.02} \left(1 - \left(\frac{1.02}{1.07}\right)^{35}\right)
\]

Solving for f, the fraction of your salary that you would like to contribute:

\[
f = \frac{121,272 \times (0.07 - 0.02)}{75,000 \left(1 - \left(\frac{1.02}{1.07}\right)^{35}\right)} = 9.948%
\]

So you would contribute approximately 10% of your salary. This amounts to $7,500 in the first year, which is lower than the plan in the prior problem.