**Solutions to Additional Problems on Risk, Betas, and WACC**

**Expected Value, Variance, Covariance, and Correlation**

1. E (Rx) = .2 (8) + .4 (12) + .3 (0) + .1 (-4) = 6%

σ2x = .2 (8-6)2 + .4 (12-6)2 + .3 (0-6)2 + .1 (-4-6)2 = 36%2

σx = 6

E (Ry) = .2 (0) + .4 (-2) + .3 (6) + .1 (20) = 3%

σ2y = .2 (0-3)2 + .4 (-2-3)2 + .3 (6-3)2 + .1 (20-3)2 = 43.4%2

σy = 6.59

σxy = .2 (2) (-3) + .4 (6) (-5) + .3 (-6) (3) + .1 (-10) (17) = -35.6



1. E(RP) = 0.5(12%) + 0.5(16%) = 14%

To find the standard deviation you need to first find the variance of the portfolio.

For the portfolio with a correlation = -1 the variance is

σP2 = (1/2)2(16%)2 + (1/2)2(26%)2 + 2(1/2)1/2)(16%)(26%)(-1) = 25%2 or in decimal form this is 0.0025. The standard deviation is the square root of the variance: σP = 5%.

For the portfolio with a correlation = 0.5 the variance is

σP2 = (1/2)2(16%)2 + (1/2)2(26%)2 + 2(1/2)1/2)(16%)(26%)(0.5) = 337%2 or in decimal form this is 0.0337. The standard deviation is the square root of the variance: σP = 18.36%.

The remaining portfolio standard deviations are:

Correlation = -0.5, Standard Deviation = 11.36%.

Correlation = 0, Standard Deviation = 15.26%.

Correlation = 1.0, Standard Deviation = 21.00%.

**Betas and WACC**

1. Think about what determines Beta. Is it the volatility of the stock or something else?
2. For this problem:

a. E(REquity) = 7% + 1.53(12% - 7%) = 14.65%

1. WACC = 14.65%(2/5) + 9%(1 - 0.26)(3/5) = 9.86%
2. a. Unlevered Beta = 0.95(1500/3200) = 0.45

b. Since the asset Beta is 0.45 then the equity beta of 0.95 can be broken down into business risk (0.45) which is the underlying risk of the company and financial risk (0.50) which is the additional risk that debt adds to the risk of equity.

1. a. E(R) = 3% + 1.2(8.5%) = 13.2%

b. Expected Price Appreciation over the next year = 13.2% - ($2.5)/$50 = 8.2%. Remember that the dividend is part of your return and total return on a stock is equal to the percentage change in stock price plus the dividend yield ($2.5/$50). The expected price one year from today is $50(1.082) = $54.10.

c. The return on the market last year was -8% + 3% = -5% (again remember the total return on a stock is the percentage change in stock price plus dividend yield). Given a beta of 1.2 then if the market is down by 5% the returns on the company should be down by 1.2(-5%) = -6% (according to the CAPM). Since HeavyTech paid a dividend of $2 last year the dividend yield for last year is then $2/$54 = 3.7%. Then in order to get a total -6% return on HeavyTech the price of HeavvTech would have to fall by 6% + 3.7% or 9.7% (notice that -9.7% + 3.7% = -6%).

d. Actual returns over last year (50-54+2)/54 = -3.7%.

e. Unlevered Beta = 1.2×($100/$150) = 0.08.

1. a. Unlevered Beta for Novel = 1.5 and for WordPerfect = 1.3. The Unlevered Beta for the combined firm is 1.5×(2/3) + 1.3×(1/3) = 1.43. Now this would be the Beta for the combined firm with no debt.

b. If the deal is financed with debt then the new debt to equity ratio is ½. If Novel had to borrow $1 billion to purchase WordPerfect then the combined firm would have $2 billion in equity and $1 billion in debt. So the new equity beta = 1.43×(1 + ½) = 2.15.

1. a. The Beta of Equity for HP = 1.1(2/8) + 1.5(2/8) + 2(1/8) + 1(3/8) = 1.275. This might not equal the Beta from a regression because the regression estimates the Beta with error.

b. Cost of Equity for HP = 7.5% + 1.275(5.5%) = 14.51%.

Mainframe Cost of Equity = 7.5% + 1.1(5.5%) = 13.55%.

Personal Computer Cost of Equity = 7.5% + 1.5(5.5%) = 15.75%.

Software Cost of Equity = 18.50%.

Printer Division Cost of Equity = 13.00%

1. Assuming that leverage is proportionally spread across the divisions based on the size of the division. What this means is that since the Mainframe business is ¼ of the total value of equity (2/(2+2+1+3) = ¼) then the amount of debt in that division would be ¼ of $1 billion or 0.25 billion. For PCs this would be the same. For software it would be 1/8 × $1B = 0.125 billion and for Printers it would be 0.375 billion. Now we have enough information to unlever the divisional betas. For the Mainframe division the unlevered beta (i.e., asset beta) would be 1.1×(2/2.25) = 0.98. For PCs = 1.5×(2/2.25) = 1.33. See the table below for the rest.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Division | Beta | Unlevered Beta | Value of Equity | Ascribed Debt | Firm Value (Division) |
| Mainframe | 1.1 | 0.98 | 2 | 0.25 | 2.25 |
| PCs | 1.5 | 1.33 | 2 | 0.25 | 2.25 |
| Software | 2 | 1.78 | 1 | 0.125 | 1.125 |
| Printer | 1 | 0.89 | 3 | 0.375 | 3.375 |

Since they sold off the mainframe division and paid out the proceeds in cash the value of HP is the value of the equity in all the remaining division 2 + 1 + 3 = 6 plus the value of the debt left. To make life easier let’s assume that they also retire the amount of debt associated with the Mainframe division which was 0.25 billion. So they retire 0.25 billion in debt leaving them with a total of 0.75 billion (you can make any assumption you want this seems to be the easiest to handle). So the value of the firm is now6.75. So the unlevered beta for the firm is

Unlevered Beta (Asset Beta) = 1.33×(2.25/6.75) + 1.78×(1.125/6.75) + 0.89×(3.375/6.75) = 1.19.

Using the formula that βE = βA×(1 + D/E) then the new Equity Beta = 1.19(1+0.75/6.75) = 1.32.

1. Remember that DOL is often defined as the (% change in operating income)/(% change in sales). So we then have the following:

|  |  |  |
| --- | --- | --- |
| **Firm** | **Operating Leverage** | **Beta** |
| PharmaCorp | 0.93 = 25%/27% | 1.00 |
| SynerCorp | 1.28 = 32%/25% | 1.15 |
| BioMed | 1.57 | 1.30 |
| Safemed | 1.90 | 1.40 |

Operating income is usually EBIT or some variation. So if DOL is high or operating leverage is high then EBIT fluctuates a lot with sales. Typically this means firms with higher operating leverage have high fixed costs. So when a firm has high operating leverage profits are typically going to fluctuate a lot more with sales than if operating leverage is low. We would expect then that Betas will be higher (lower) in firms with higher (lower) operating leverage.

1. a. If you were going to calculate the expected return over the next year you probably want to use the short term risk free rate in the CAPM. In this case E(R) = 4.8% + 1.65(5.5%) = 13.86%.

b. Yes I would use the longer term risk free rate of 6.4% in CAPM (but same MRP of 5.5%).

c. If the firm has 265 million shares outstanding and a stock price of $30 per share then the market value of equity is $7.95 billion. If the D/E ratio is 3% then the value of debt is 3% × 7.95 billion = $238.5 million. So the Unlevered Asset Beta = 1.65(7.95/8.1885) = 1.60. If the firm issues new debt of $2 billion then the total debt is $2.2385 billion. The new Levered Equity Beta is then 1.60(1 + D/E) = 1.60(1 + 2.2385/7.95) = 2.05.

1. a. Unlevered Asset Beta for Southwest in 1995 = 0.90(1/2) = 0.45. Unlevered Beta for the Media Business = 1.2(2/3)=0.8. Unlevered Beta in 1999 = 0.30(0.8) + 0.70(0.45) = 0.56. Equity Beta for 1999 = 0.56(1 + 1) = 1.12.

b. If it finances the media business with a D/E of 50% then the overall D/E ratio is 1(0.7) + 0.5(0.3) = 0.85. One way to think about this is that for every dollar of debt in the media division they have $2 of equity for a D/E of ½. For the company before the media division for every $1 of debt there was a $1 of equity for a D/E of 1. Now the media division is 30% of the firm and the other division(s) is 70%. Beta with that ratio is 0.56(1+0.85) = 1.04.

1. a. No, it is not unusual. As firms grow and become larger, they generally become more diversified and less risky (but of course not always).

b. I would expect it to continue since Adobe still has a beta well above one.

c. I would expect it to stop as the beta approaches one.